Using a computer, MATLAB and Excel, consider the following problems.

1. In MATLAB, enter the following commands:
   - `a = 1e-3`
   - `b = 0.001`
   - `c = 10^0;`
   - `d = 1`
   - `e = c+a`
   - `f = e-b`
   - `g = 1/f`

   (a) What is different about the statement declaring `c`?
   (b) What is the exact (theoretical) result for `g`?
   (c) What is the exact result for `g-1`?
   (d) What does MATLAB provide for `g-1`?
   (e) What is happening here?

2. In MATLAB enter the following commands and think about what they signify:
   - `eps`
   - `help eps` – or – `doc eps`

3. In computational work, `eps` often is used as a synonym for machine precision, or the smallest value ϵ for which $1 + \epsilon > 1$. Estimate your own value of `eps`
   (a) using a `for` loop in MATLAB;
   (b) using a `while` loop in MATLAB;
   (c) compare the two techniques and the value for `eps` given by MATLAB directly.

4. Using an Excel spreadsheet and formulae in cells, estimate `eps`.

5. You should be familiar with solutions to equations of the form $ax^2 + bx + c = 0$ as $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$. Using MATLAB and various coefficients a, b and c do the following:

   (a) Convince yourself (numerically) that $x = \frac{2c}{-b \pm \sqrt{b^2-4ac}}$ are also solutions to the quadratic equation.

   (b) Similarly, verify that $x = \frac{b+\text{sgn}(b)\sqrt{b^2-4ac}}{-2a}$, $\frac{-2c}{b+\text{sgn}(b)\sqrt{b^2-4ac}}$ are also solutions.[1]

   (c) Is there an advantage to the second form of the solution?

   (d) Is there an advantage to the third form of the solution?

   (e) Can MATLAB handle the case where $b^2 < 4ac$?

6. Write a function in MATLAB that solves a quadratic equation for a given set of coefficients a, b and c.

7. A soccer ball is placed on a level surface, a distance d_1 from another parallel surface that is some height h above/below it. Your goal is to kick the soccer ball and smash a Styrofoam cup on the second surface a distance d_2 from the boundary between the two surfaces. Using a special set of parameters $d_1 = 0, h = 0$ and an appropriate choice for d_2:

   (a) Use MATLAB or Excel to find the initial conditions (angle of your kick θ_o and velocity v_o) required to strike the cup.

   (b) How precise is your answer?

   (c) Is your solution unique?

   (d) What if $d_1 \neq 0$ and $h \neq 0$? Are there any special considerations that must be taken into account in order to arrive at a correct solution (just discuss this, don't bother to code it up just yet).

[1] Here sgn(x) is the signum function, defined as 1 if $x \geq 0$ and -1 if $x < 0$. In MATLAB it is called via `sign(x)`. What does MATLAB give for `sign(0)`? Does this discrepancy matter in this third form of the solutions?