Complete the following problems on a separate sheet(s) of paper. Your solution is due at the beginning of the final examination session, 1 pm on Friday, 8 May. The examination is worth 20 points (the point breakdown is given for each problem). The use of books, notes, computers, etc., is allowed for this examination; however, you are expected to do your own work. Be sure to complete both sides of the examination paper.

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1. Consider the following, hopefully familiar, equation of motion:

$$\frac{dv}{dt} = -g - bt \ .$$

   (a) [4] Given that $b = 1.0$, fill in the table by carrying out two Euler steps for v to the indicated precision.

t	$\frac{dv}{dt}$	v
0.00	-10.00	0.000
0.05		
0.10		

   (b) [1] As $t \to \infty$, describe the behavior of v.

   (c) [1] Will the Euler method be sufficient to study such a problem? Explain.

   (d) [1] Describe a physical situation that corresponds to the initial conditions given above.

2. In studying atmospheric acoustics, you're looking at the wave equation to model infrasound ($f \leq 16$ Hz) propagation. The wave speed in the air you're concerned with is 330 m/s and the signal of interest is centered at 3 Hz.

   (a) [1] What is the wavelength of the center frequency of this sound?

   (b) [1] Given this wavelength, give a reasonable choice for Δr, the spatial grid size. Explain your choice.

   (c) [1] Apply the Courant-Friedrichs-Lewy condition to determine an appropriate choice (or range of choices) for Δt.

3. You're studying the rotation of a disc governed by the model equation

$$\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2 .$$

You have taken N observations of identically prepared experiments, forming the set of ordered pairs $\{(t, \theta_f)\}_N$. Your goal is to find the numerical value of the angular acceleration (or deceleration in this case) α. Use the normal equation formulation of linear least squares to set up expressions for

   (a) [1] the design matrix $\mathbb{M}$;
   (b) [1] the parameter vector $|a\rangle$;
   (c) [1] and the dependent variable vector $|b\rangle$.

4. [1] Laplace's equation in electrostatics is elliptical in form, so that the system it describes is in a steady state. In this course we numerically solved the diffusion equation

$$\frac{\partial V}{\partial t} = D\left(\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2}\right)$$

in order to find a solution to Laplace's equation. Describe how the diffusion equation is used in the Jacobi relaxation method to accomplish this.

5. [6] In our study of the time-independent Schrödinger equation, we found an approximate numerical solution in the form

$$\psi_{n+1} = 2\psi_n - \psi_{n-1} - 2\left(\Delta x\right)^2 \left(E - V_n\right)\psi_n .$$

Describe the six variables in this approximation.